

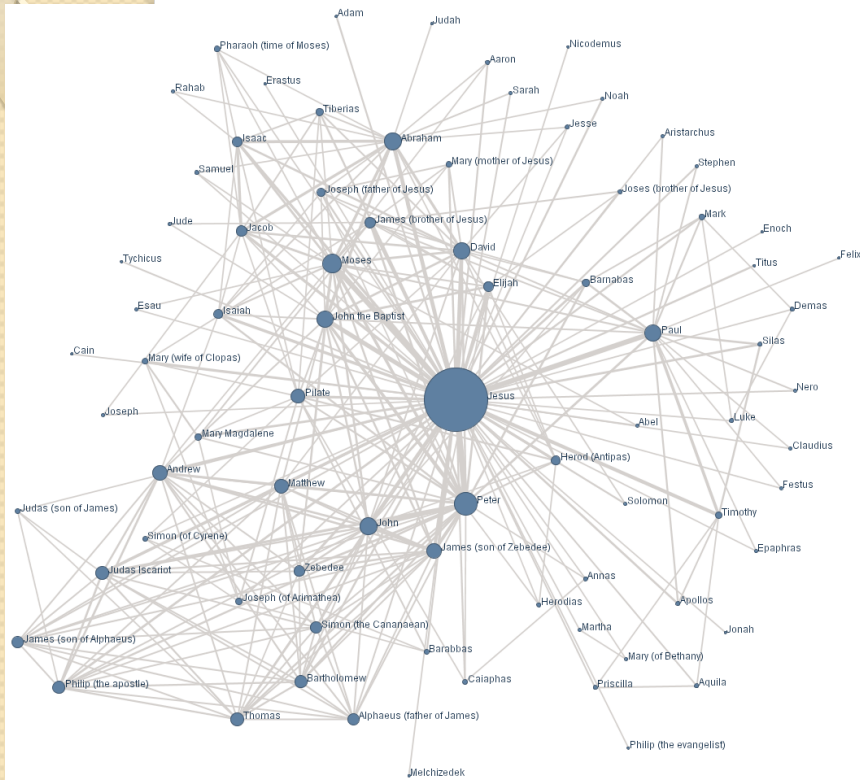


Recovering Temporally Rewiring Networks: A Modal-based Approach

Presented by Chao Ji

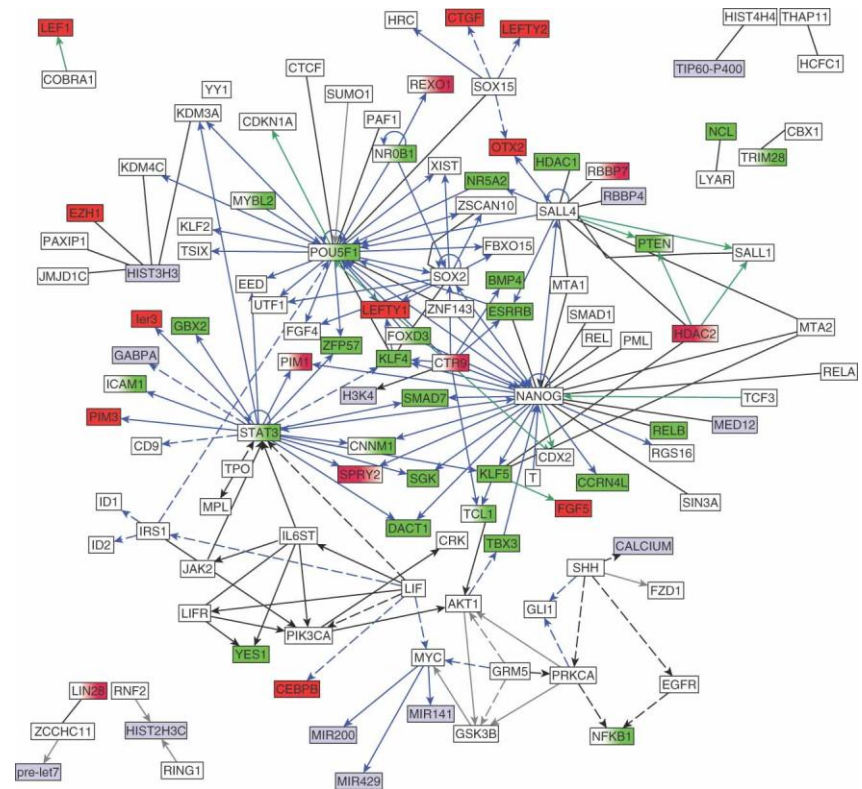
Guo et al. in ICML 07'

Examples of Networks



Social network

Gene regulatory network





Static, Observable vs Dynamic, latent Networks

- The interactions between nodes may evolve over time
 - More edges may form in the social network as we know more friends
 - The activate/suppress relationship in gene regulatory networks may be turned on/off under different conditions
- The interactions between nodes may not be observable
 - Infer the hidden network topology from observable node attributes

A Classical Model for Static Networks

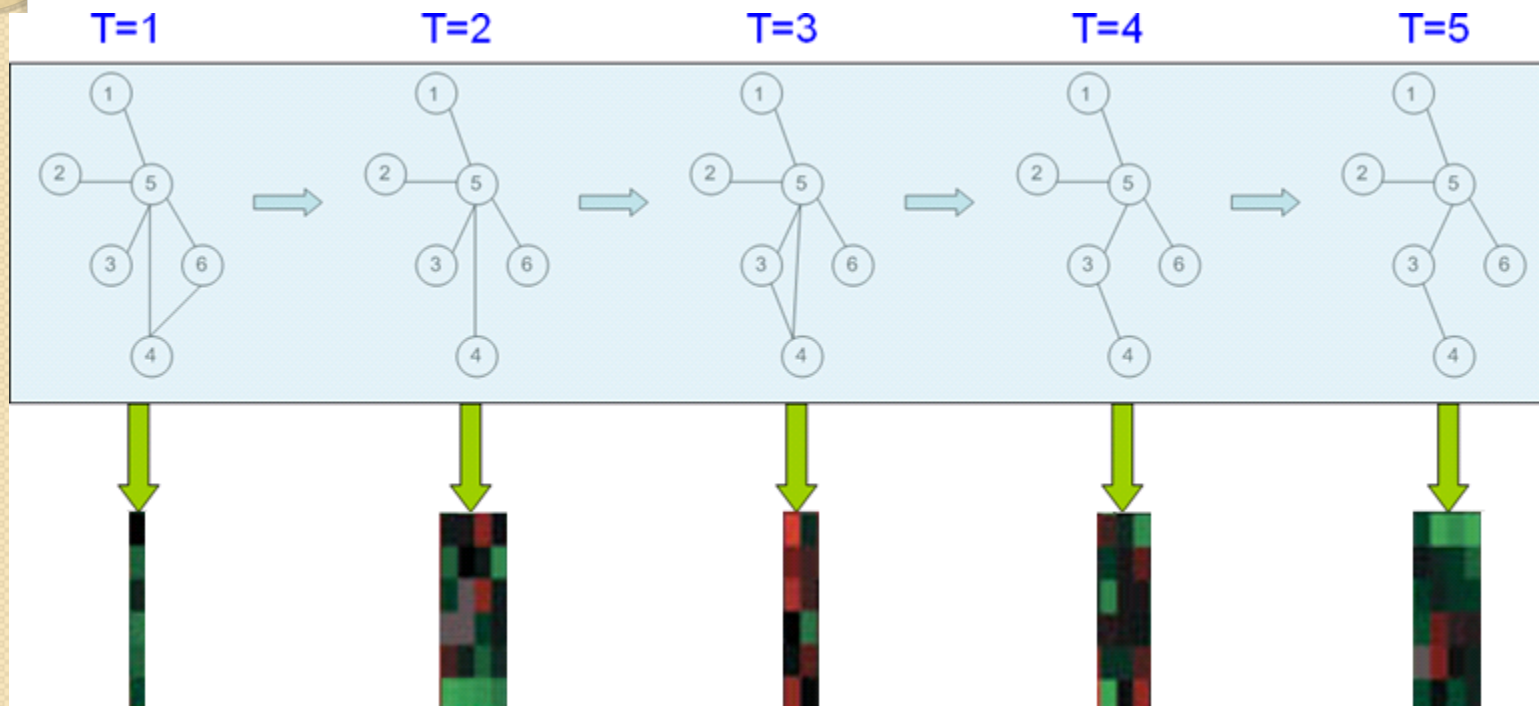
- Exponential Random Graph Models (ERGM)

$$P(A) = \frac{1}{Z(\boldsymbol{\theta})} \exp \left\{ \sum_c \theta_c \Phi_c(A) \right\}$$

Modeling the distribution of network topology

Rewiring networks

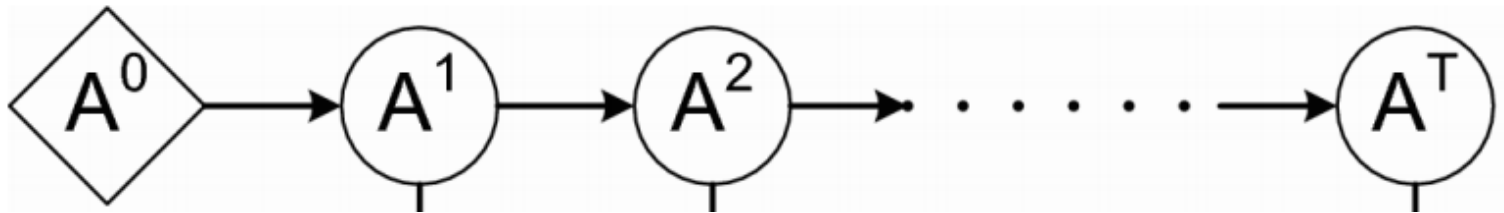
- Network rewires over discrete time steps



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- Data: time series of node attributes (e.g. levels of gene expressions)
 - Goal: recover the unobserved temporal rewiring networks from samples of node states
 - Assumption
 - fixed set of nodes
 - Markov process

Extending ERGM to tERGM

- temporal ERGM for modeling a observed sequence of networks



Extending ERGM to tERGM

Markovian Process:

$$P(\mathbf{A}^1) \prod_{t=2}^T P(\mathbf{A}^t | \mathbf{A}^{t-1})$$

$$P(\mathbf{A}^t | \mathbf{A}^{t-1}) = \frac{1}{Z(\boldsymbol{\theta}, \mathbf{A}^{t-1})} \exp \{ \boldsymbol{\theta}' \boldsymbol{\Psi}(\mathbf{A}^t, \mathbf{A}^{t-1}) \}$$

Potential functions or *features*:

“Density” $\Psi_1(\mathbf{A}^t, \mathbf{A}^{t-1}) = \sum_{ij} A_{ij}^t,$

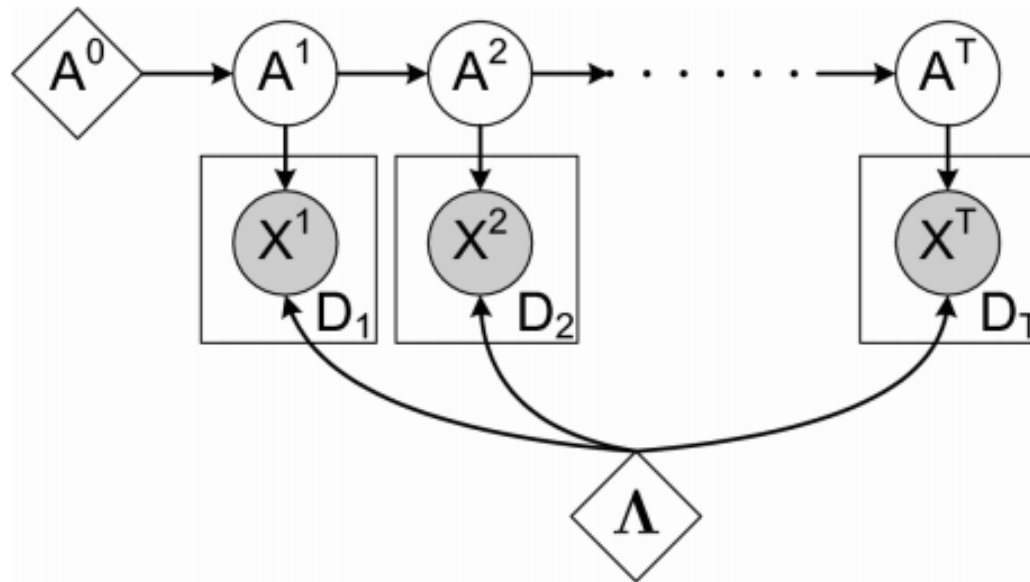
“Stability” $\Psi_2(\mathbf{A}^t, \mathbf{A}^{t-1}) = \sum_{ij} \mathbb{I}(A_{ij}^t = A_{ij}^{t-1}),$

“Transitivity” $\Psi_3(\mathbf{A}^t, \mathbf{A}^{t-1}) = \frac{\sum_{ijk} A_{ij}^t A_{ik}^{t-1} A_{kj}^{t-1}}{\sum_{ijk} A_{ik}^{t-1} A_{kj}^{t-1}},$

$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$:Parameters for adjusting the strength of each of the three features

Extending tERGM to htERGM

- hidden temporal ERGM



Extending tERGM to htERGM

$\mathbf{x}^t = \{\mathbf{x}_{1:N}^t\}$ denote observed attributes over all N nodes of a network A^t at timestep t

Joint probability
$$P(\mathbf{A}^{1:T}, \mathbf{x}^{1:T} | \mathbf{A}^0, \Lambda) = \prod_{t=1}^T P(\mathbf{A}^t | \mathbf{A}^{t-1}) P(\mathbf{x}^t | \mathbf{A}^t, \Lambda).$$

Emission model
$$P(\mathbf{x}^t | \mathbf{A}^t) = \frac{1}{Z(\eta, \mathbf{A}^t)} \exp \left\{ \eta' \sum_{ij} \Phi(x_i^t, x_j^t, A_{ij}^t, \Lambda_{ij}) \right\}$$

feature function
$$\Phi(x_i^t, x_j^t, A_{ij}, \Lambda_{ij}) = A_{ij}^t \Lambda_{ij} (2\mathbb{I}(x_i^t = x_j^t) - 1)$$

Extending tERGM to htERGM

feature function $\Phi(x_i^t, x_j^t, A_{ij}, \Lambda_{ij}) = A_{ij}^t \Lambda_{ij} (2\mathbb{I}(x_i^t = x_j^t) - 1)$

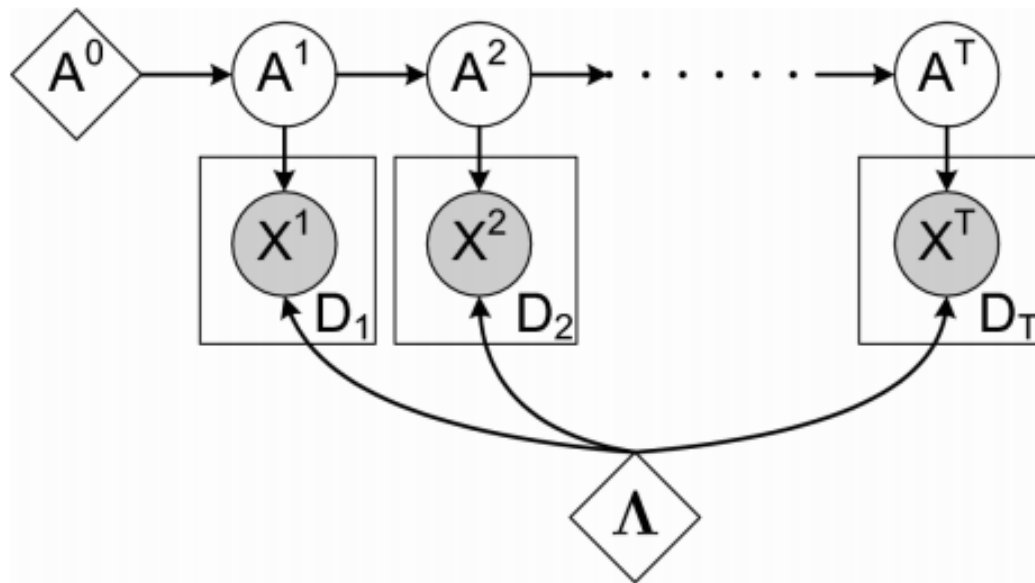
$\Phi = 0$ if $A_{ij} = 0$, otherwise it depends on the sign of Λ_{ij} and x_i and x_j

x_i^t	x_j^t	$\Lambda_{ij} = 1$	$\Lambda_{ij} = -1$
(x)	(x)	$\Phi = 0$	$\Phi = 0$
(0) — (0)		$\Phi = 1$	$\Phi = -1$
(0) — (1)		$\Phi = -1$	$\Phi = 1$
(1) — (0)		$\Phi = -1$	$\Phi = 1$
(1) — (1)		$\Phi = 1$	$\Phi = -1$

Inference

Sampling formula $A_{ij}^t \sim \text{Bernoulli}(1/(1 + \exp(-\mu_{ij}^t)))$

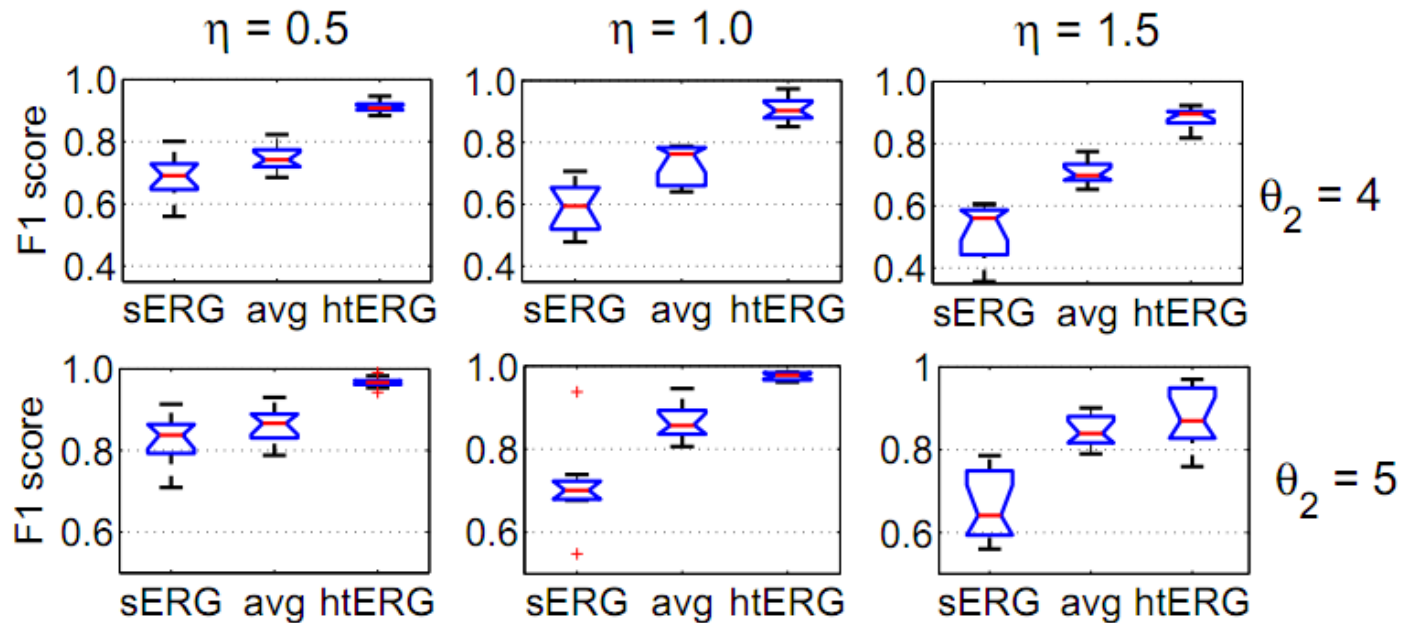
Log-odds: $\mu_{ij}^t = \log \frac{P(A_{ij}^t = 1 | \mathbf{A}^{t-1}, \mathbf{A}_{-ij}^t, \mathbf{A}^{t+1}, \mathbf{x}^t)}{P(A_{ij}^t = 0 | \mathbf{A}^{t-1}, \mathbf{A}_{-ij}^t, \mathbf{A}^{t+1}, \mathbf{x}^t)}$



Results from simulated data

- $N = 10$ nodes with 14 edges
- Total number of time steps $T = 50$

Results from simulated data



Results from simulated data

